SEAA 3012 CIVIL ENGINEERING LABORATORY II · LAB S2

Influence Lines

Cantilever (Gerber) bridge model

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Today

1. Why influence lines? What they are, and what they are not
2. The laboratory model and why it is determinate
3. Deriving the six influence lines from statics
4. The Müller–Breslau shortcut
5. Using an influence line: one load, a pair, a distributed load
6. The experiment: data reduction, errors, what to report



Loads that move

A floor load stays put. A lorry on a bridge, a crane on its rail, a train on a viaduct do not.

The designer needs to know **where** the load must stand to produce

- the largest reaction at a pier (bearing design, uplift)
- the largest shear at a section
- the largest moment at midspan

Definition

The **influence line** of a response R at a fixed point is the graph of R against the position x of a **unit** load moving over the structure: ordinate $\eta_R(x)$.

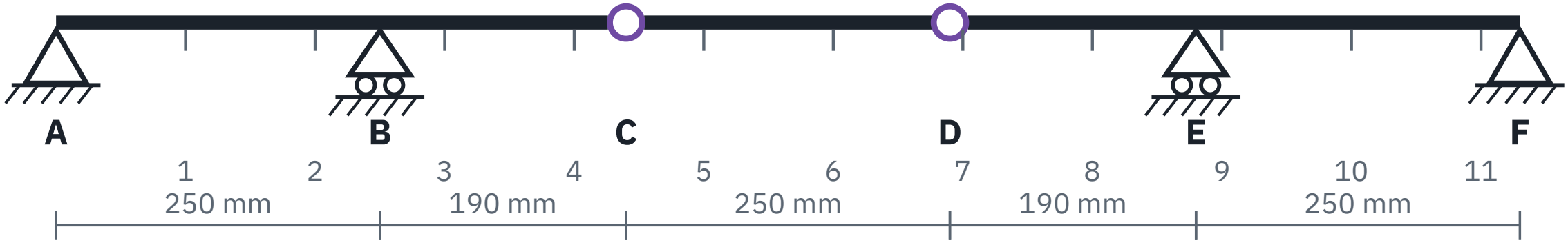


Influence line versus bending-moment diagram

	SFD / BMD	Influence line
Fixed	the loads	the point where R is read
Varies along x	the section where V, M are read	the position of the unit load
Ordinate means	V or M at that section	R at the fixed point, load at x

The structure is linear elastic, so effects add: once $\eta_R(x)$ is known for a unit load, **any** load pattern follows by multiplication and addition.

A Gerber bridge, 1130 mm long



- ABC: pin A, roller B, cantilever BC
- CD: suspended span on **hinges** C and D
- DEF: mirror image of ABC
- Gauges A, B, E, F: support reactions
- Gauges C, D: hinge forces
- Halts 1–11: where the mass is placed

Why it is statically determinate

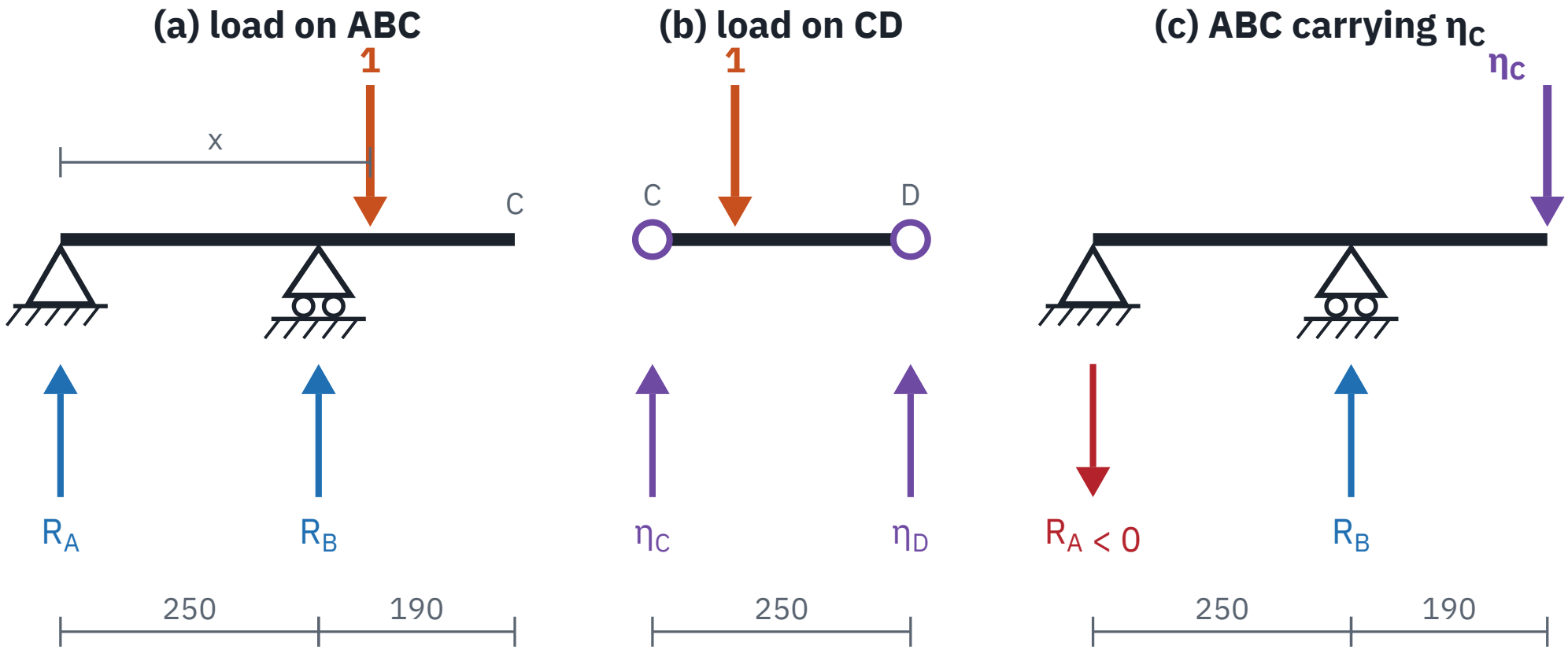
Unknowns: R_A, R_B, R_E, R_F: four.

Equations: $\sum F_y = 0$ and $\sum M = 0$ for the whole model, plus $M = 0$ at hinge C and at hinge D: four.

Consequences

- every reaction from equilibrium; EI never appears
- every influence line is made of **straight segments**, kinked only at hinges
- reactions positive up; R_C, R_D positive pressing down on the tip

One idea, three times: take moments about a support



Order matters: analyse the suspended span first; it is the only member whose loads reach the others.

Step 1 · Load on beam ABC

Moments about B (hinge C unloaded):

$$\eta_A(x) = \frac{250 - x}{250}, \qquad \eta_B(x) = \frac{x}{250} \qquad (0 \le x \le 440)$$

- Nothing stops at B: past B the lever arm $(250 - x)$ is **negative**
- $\eta_A < 0$: support A must *hold the beam down*
- $\eta_B > 1$: B carries the load *and* the pull-down at A

At the tip C

$\eta_A(440) = -0.76$
$\eta_B(440) = 1.76$



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Steps 2 and 3 · Load on the suspended span

Step 2. CD is simply supported on the hinges:

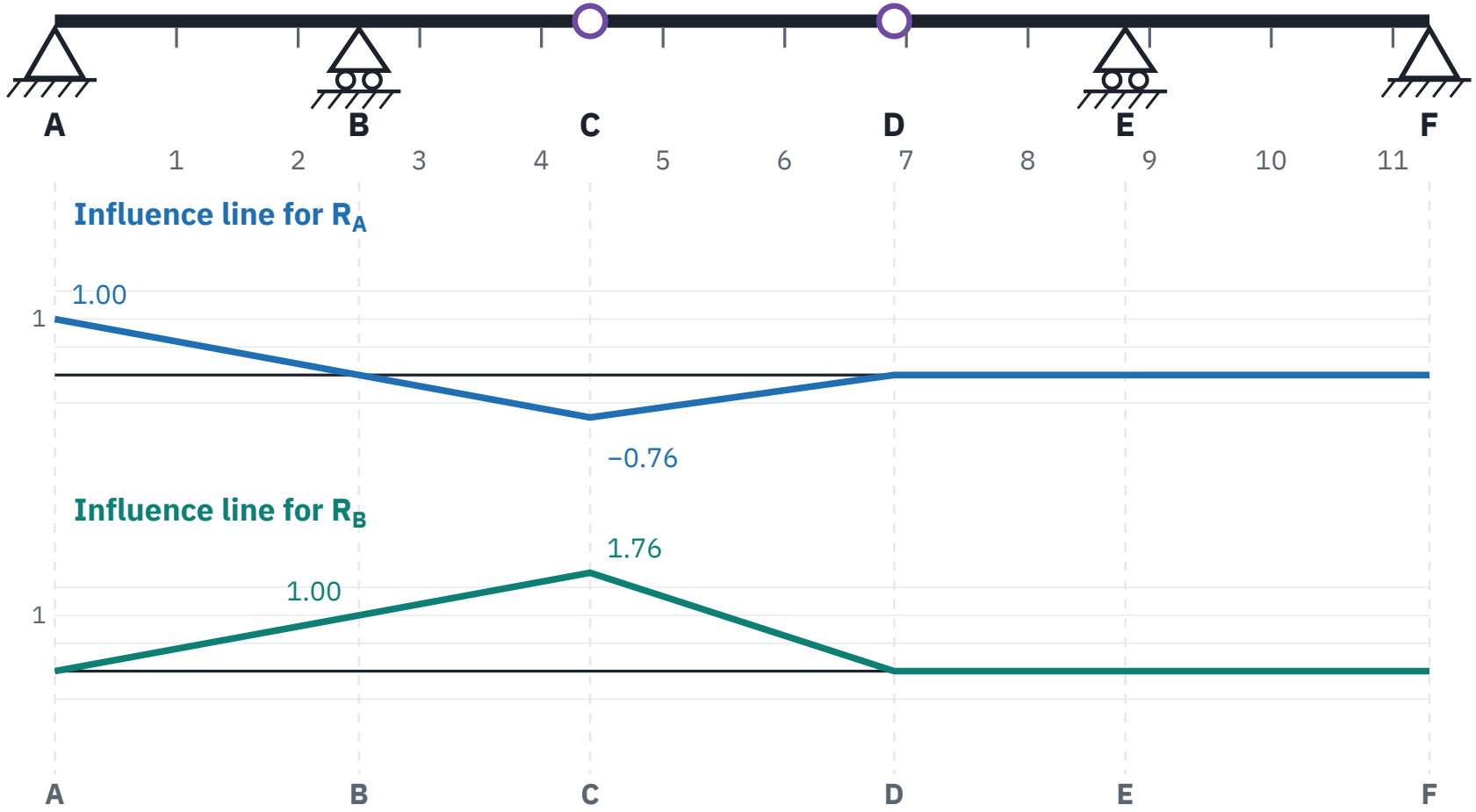
$$\eta_C(x) = \frac{690 - x}{250}, \qquad \eta_D(x) = \frac{x - 440}{250}$$

Step 3. ABC carries only η_C at its tip:

$$\eta_A(x) = -0.76\,\eta_C(x), \qquad \eta_B(x) = 1.76\,\eta_C(x)$$

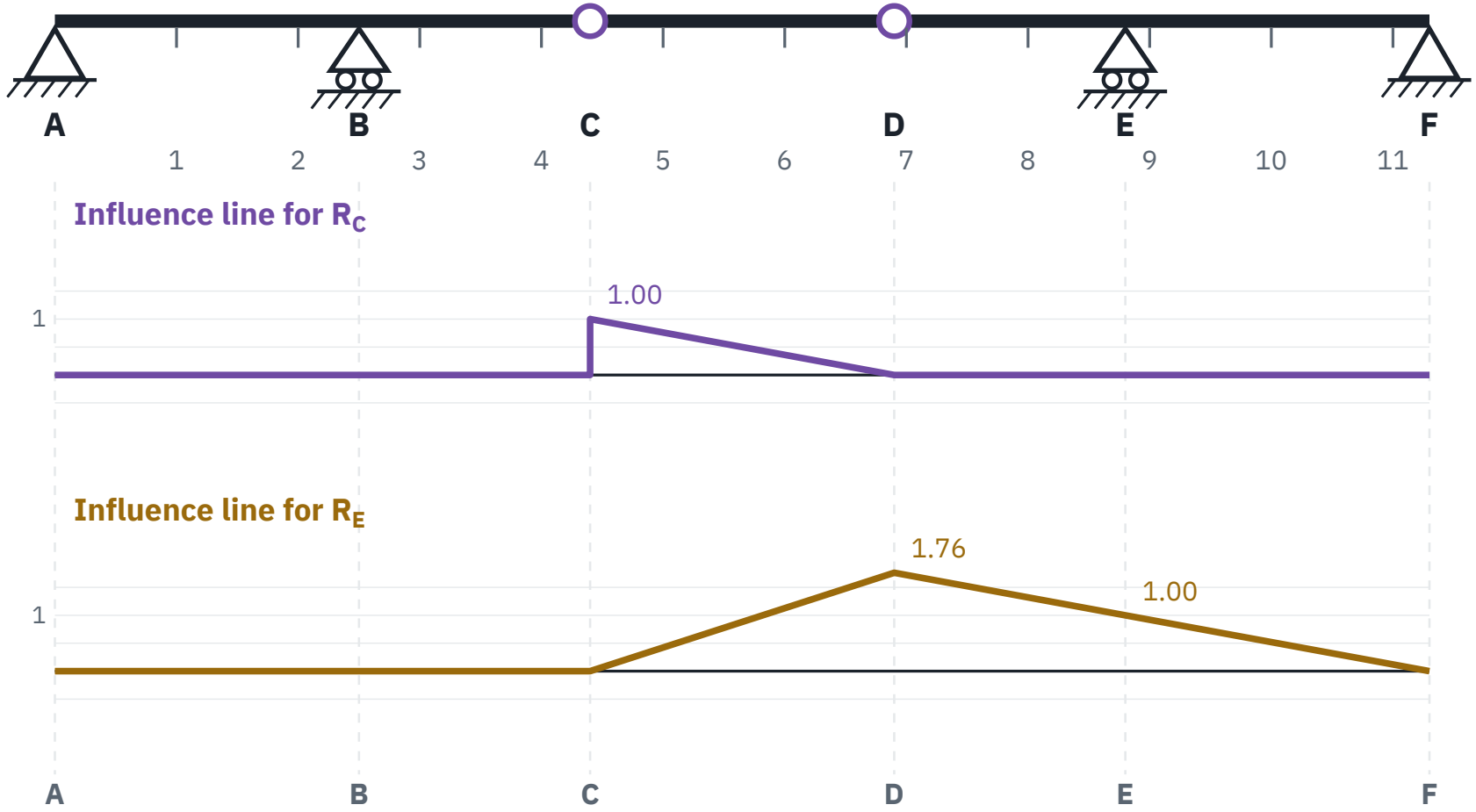
Continuous at C (-0.76 from both sides); zero at D.

The influence lines, under the bridge



Same horizontal scale: each halt sits directly above its ordinate. Check: $\eta_A + \eta_B + \eta_E + \eta_F = 1$ at every x.

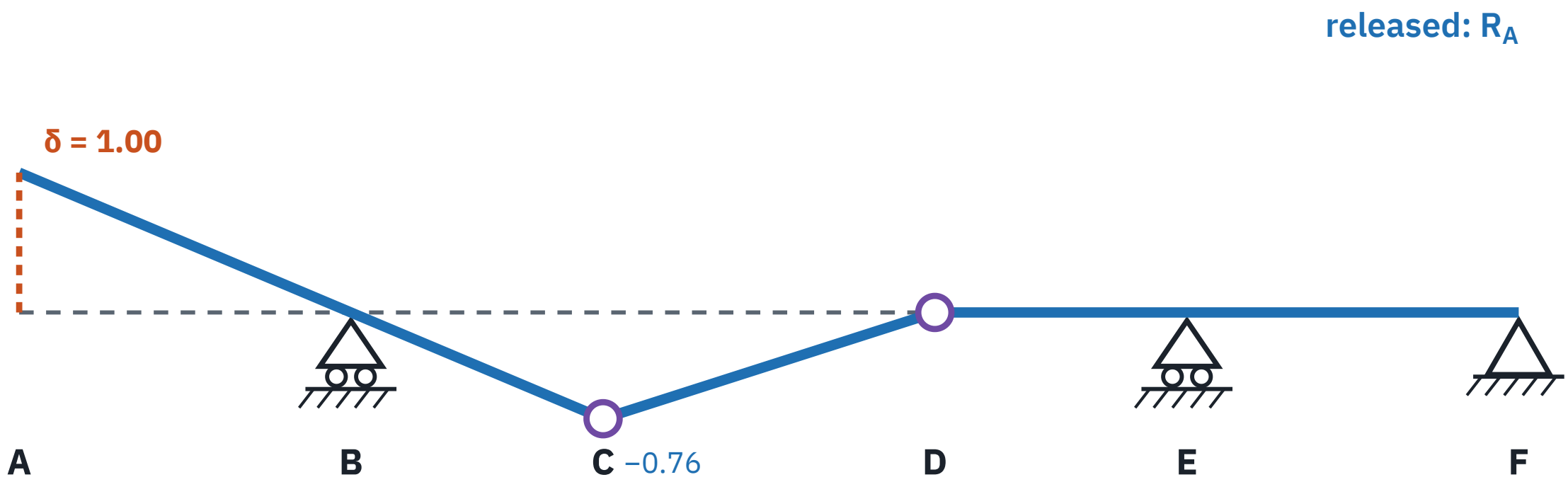
Hinge force and the right-hand side



R_C acts only for loads on CD; R_D, R_F mirror R_C, R_A. B and E carry up to 1.76 times a load on the tips.

Müller–Breslau: the engineer's shortcut

Release the restraint, impose a unit displacement: the deflected shape is the influence line.



For R_A: ABC rotates about B as a rigid bar, C drops 0.76, CD follows, DEF stays still. Same line, no algebra.

A mass stands at halt 8, between D and E. The reaction at A is ...

- upward
- zero
- downward

Using an influence line

$$R = P\,\eta_R(x_1) \qquad R = \sum_i P_i\,\eta_R(x_i) \qquad R = w \times \text{area}$$

Critical position

All our lines are piecewise linear, so a sum over a load train is piecewise linear in its position: the extreme occurs with **one load on a vertex**.

Units

Masses in kg give reactions in kgf, which is what the gauges read (1 division = 0.05 kgf).



Worked example 1 · The compound load of Test 2

2.5 kg mass 100 mm to the right of the 5 kg mass (in front, moving right):

$$R_A(x) = 5\,\eta_A(x) + 2.5\,\eta_A(x+100)$$

- 5 kg at A: $5(1.0) + 2.5(0.6) = 6.5$ kgf
- 5 kg at C: $5(-0.76) + 2.5(-0.456) = -4.94$ kgf
- 2.5 kg at C: $5(-0.36) + 2.5(-0.76) = -3.70$ kgf

5 kg at	x (mm)	R_A (kgf)	R_B (kgf)
A	0	6.50	1.00
1	100	3.50	4.00
2	200	0.50	7.00
B	250	−1.00	8.50
3	300	−2.50	10.00
4	400	−4.44	11.34
C	440	−4.94	11.44
5	500	−3.57	8.27
6	600	−1.37	3.17



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Worked example 2 · A distributed load

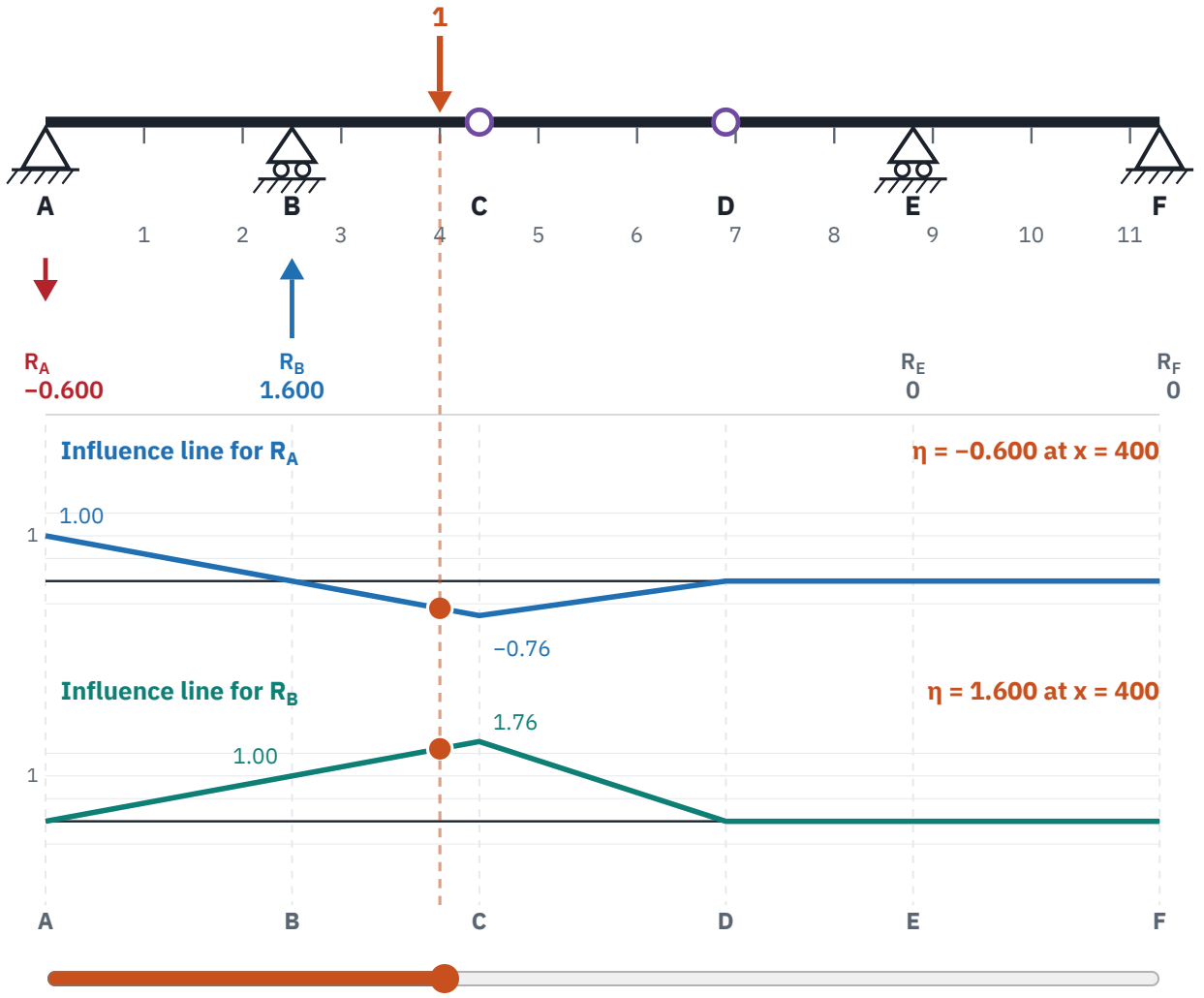
Areas under η_A: $+125$ mm (AB), -72.2 mm (BC), -95.0 mm (CD), 0 (DEF). For $w = 0.01$ kg/mm:

- over AB only: $R_A = 1.25$ kgf
- over BC and CD: $R_A = -1.67$ kgf
- over the whole model: $R_A = -0.42$ kgf: the deck **lifts** at A

Check by statics

$$R_A \cdot 250 = 250w(125) - 190w(95) - 125w(190) = -10\,550\,w \;\Rightarrow\; R_A = -42.2\,w$$

Try it live


1
A
B
C
D
E
F
1 2 3 4 5 6 7 8 9 10 11
R_A
−0.600
R_B
1.600
R_E
0
R_F
0
Influence line for R_A
η = −0.600 at x = 400
1.00
1
−0.76
Influence line for R_B
η = 1.600 at x = 400
1.76
1.00
1
A
B
C
D
E
F

The experiment

Test 1: 2.5 kg at each of 17 halts, read all six gauges.
Test 2: 5 kg + 2.5 kg pair, 100 mm apart.

Data reduction

$$\eta_{\text{test}} = \frac{\text{reading} \times 0.05}{m}, \; m = 2.5 \text{ kg}$$

Test 2: reading × 0.05 = reaction in kgf

Before leaving the laboratory

- equilibrium check: $(a + b + e + f) \times 0.05 \approx 2.5$ kgf
- re-zero the gauges between halts
- centre the mass on the halt (±5 mm matters)



How good is “good agreement”?

Source	Effect on η_A
Mass ±5 mm off the halt	±0.020
Gauge ±1 division	±0.020
Zero drift	±0.02–0.04
Combined (first two)	±0.03

Percentages mislead

±0.03 is 3 % of the ordinate at A but 15 % at halt 2, and undefined where the ordinate is zero. Compare by **absolute difference**.



Your turn

1. Explorer: the five tasks, then the five concept questions
2. Laboratory: enter your readings in the Lab tab and check equilibrium at every halt
3. After the laboratory: Exercises (hand calculation), then the report



Scan for the Explorer

Summary

Six straight-line influence lines

$\eta_A = (250 - x)/250$ on ABC, $-0.76\,\eta_C$ on CD

$\eta_B = x/250$ on ABC, $1.76\,\eta_C$ on CD

$\eta_C = (690 - x)/250$,

$\eta_D = (x - 440)/250$

η_E, η_F: mirror images

Use and report

$R = \sum P_i\,\eta_R(x_i)$; UDL: $w\times$ area

extreme: one load on a vertex

Müller–Breslau: release, displace, read

compare by absolute difference



Reading: Hibbeler, Structural Analysis, ch. 6; Kassimali, Structural Analysis, ch. 8; Laboratory Manual, experiment S2.